# Universal power-efficiency trade-off in battery charging


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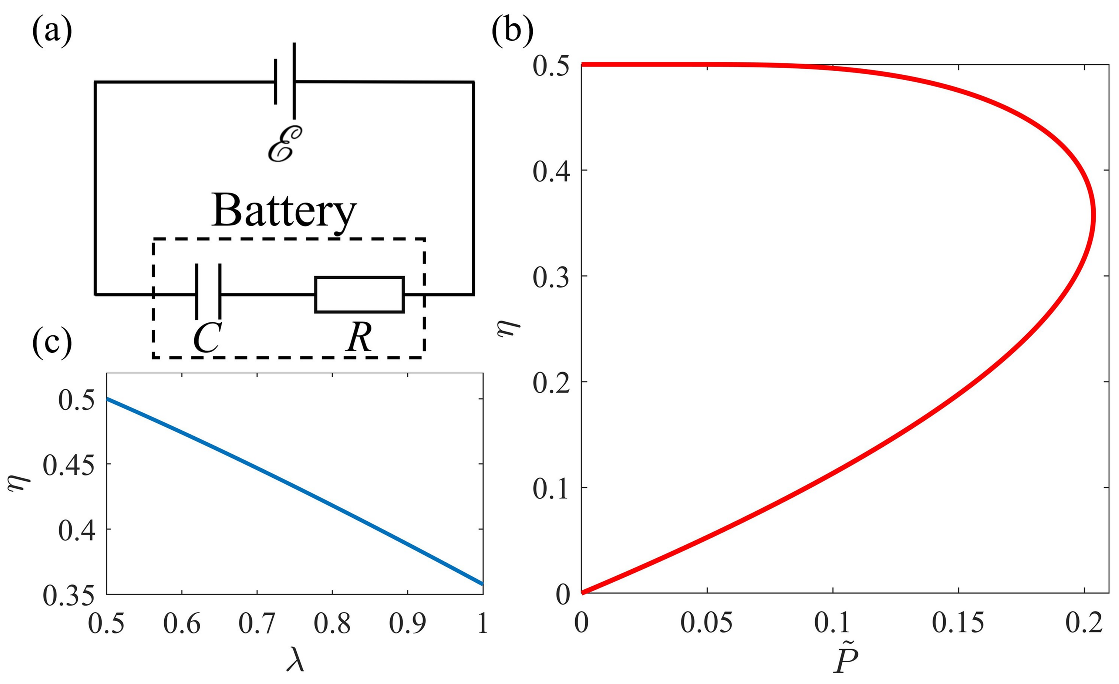
## 1. Introduction

Designing efficient and fast-charging batteries is an important goal in the field of energy, crucial for upgrading new energy vehicles and portable electronic devices such as smartphones. Here, we incorporate the concept of finite-time thermodynamics into studying the resistor-capacitor (RC) series circuit and obtain the time-dependence of charging efficiency and charging power. Through this exploration, essential thermodynamic constraints governing the charging process, including the trade-off relation between charging power and efficiency, are obtained. Moreover, we reveal the lower bound for charging time and the corresponding optimal charging strategy, and further demonstrate the power-efficiency trade-off relation in such an optimized strategy. Our findings shed new light on seeking optimal battery charging methods with nonequilibrium thermodynamics.

## 2. P-$\eta$ trade-off for CV charging




For the simplest RC circuit, the voltage and charging current of the battery are

$$V(t) = E - (E - V_1)\exp(-\frac{t}{CR})$$

$$I(t) = \frac{E - V_1}{R}\exp(-\frac{t}{CR})$$

The output energy of the source and the dissipated energy due to circuit heating are

$$W_{\text{out}}(\tilde{\tau}) = \mathcal{W}\lambda\left[1 - \exp(-\tilde{\tau})\right]$$

$$Q_{\text{diss}}(\tilde{\tau}) = \frac{\mathcal{W}\lambda^2}{2}\left[1 - \exp(-2\tilde{\tau})\right]$$

➙ **charging power & charging efficiency**:

$$P = \frac{\mathcal{W}\lambda\left[1 - \exp(-\tilde{\tau})\right]\left[1 - \lambda\frac{1+\exp(-\tilde{\tau})}{2}\right]}{CR\tilde{\tau}}$$

$$\eta = 1 - \lambda\frac{1 + \exp(-\tilde{\tau})}{2}$$

✓ **$P - \eta$ trade-off:**

$$\frac{P}{E^2R^{-1}} = \frac{2(\lambda - 1 + \eta)\eta}{\ln\left[\lambda/(2 - 2\eta - \lambda)\right]}$$

✓ **Efficiency at maximum charging power $\eta_{\text{MCP}}$:**

$$(\lambda - 1 + 2\eta)\ln\frac{\lambda}{2 - 2\eta - \lambda} - \frac{2\eta(\lambda - 1 + \eta)}{2 - 2\eta - \lambda} = 0$$

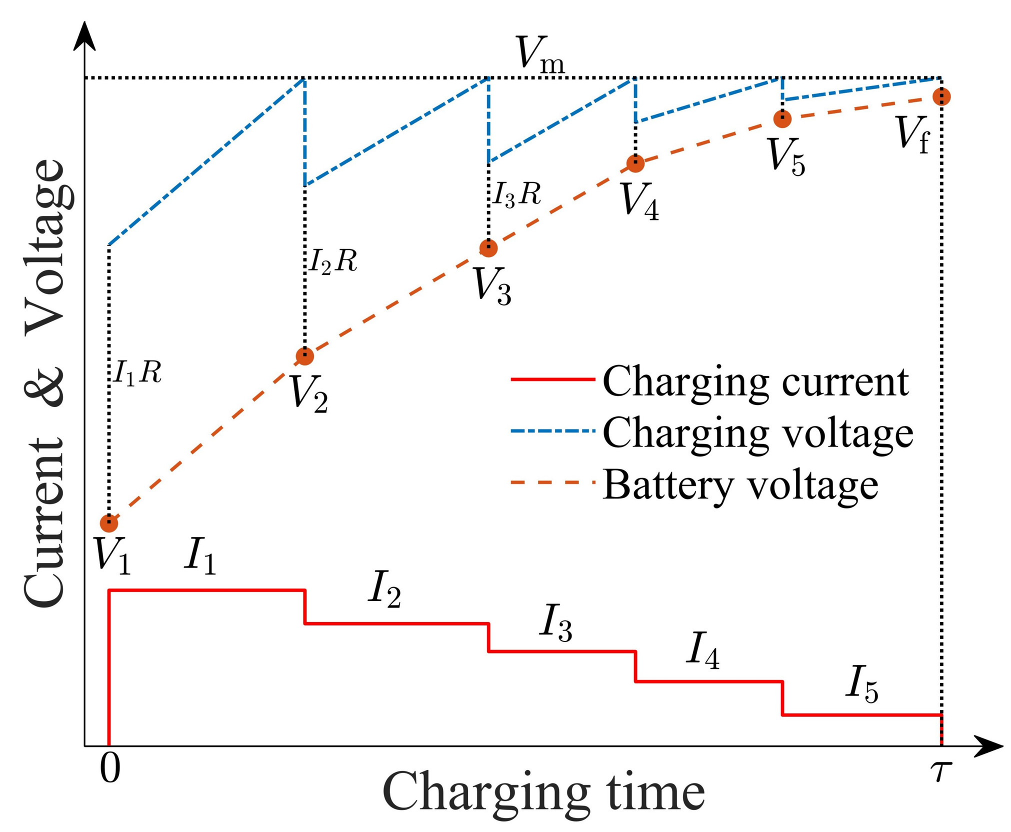
## 3. Optimization of the MSCC charging

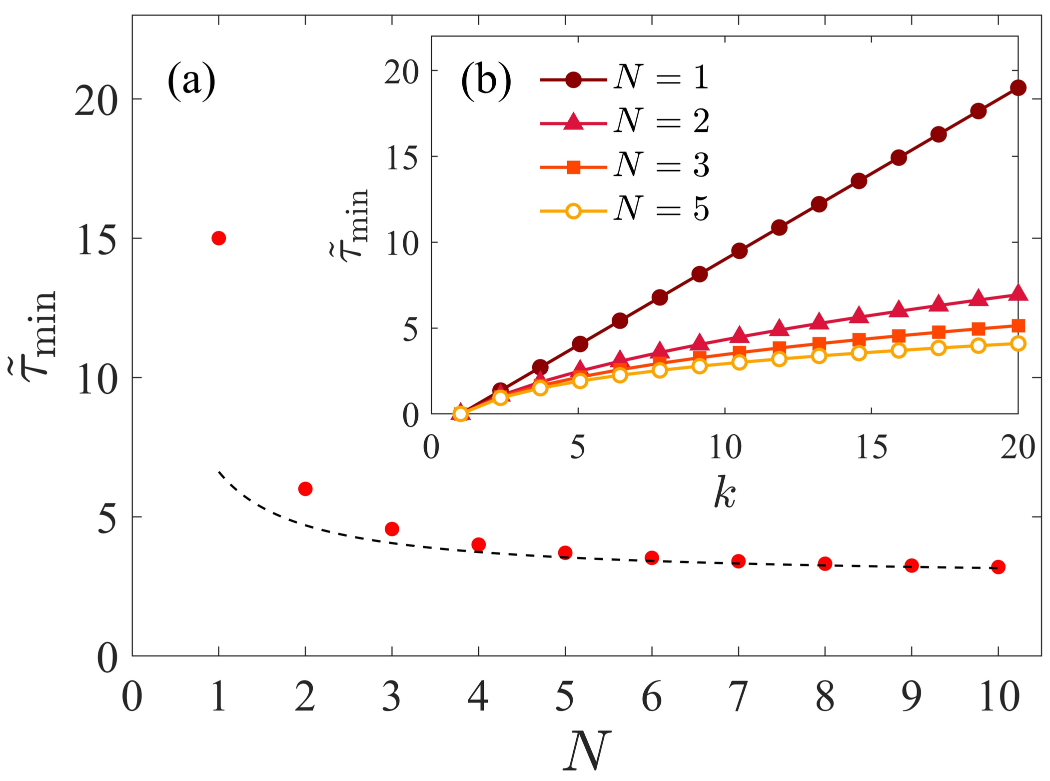
Multistage constant current (MSCC) is the most used charging strategy, and CV charging alongside CC charging emerges as the minimal case of MSCC. We take MSCC as an examle to derive **OCP, minimum charging time, and further illustrate the power-efficiency trade-off**.

- The total charging time: $\tau = \frac{C}{I_1}(V_{\text{m}} - V_1) - CR + \sum_{i=2}^{N} CR(\frac{I_{i-1}}{I_i} - 1)$

With $\partial\tau/\partial I_i = 0$ and $\partial^2\tau/\partial I_i^2 > 0$ the OCP and minimal charging time for MSCC are derived as

- $I_i = \frac{1}{R}(V_{\text{m}} - V_1)^{\frac{N-i}{N}}(V_{\text{m}} - V_{\text{f}})^{\frac{i}{N}}$
- $\tau_{\min} = NCR(k^{\frac{1}{N}} - 1)$



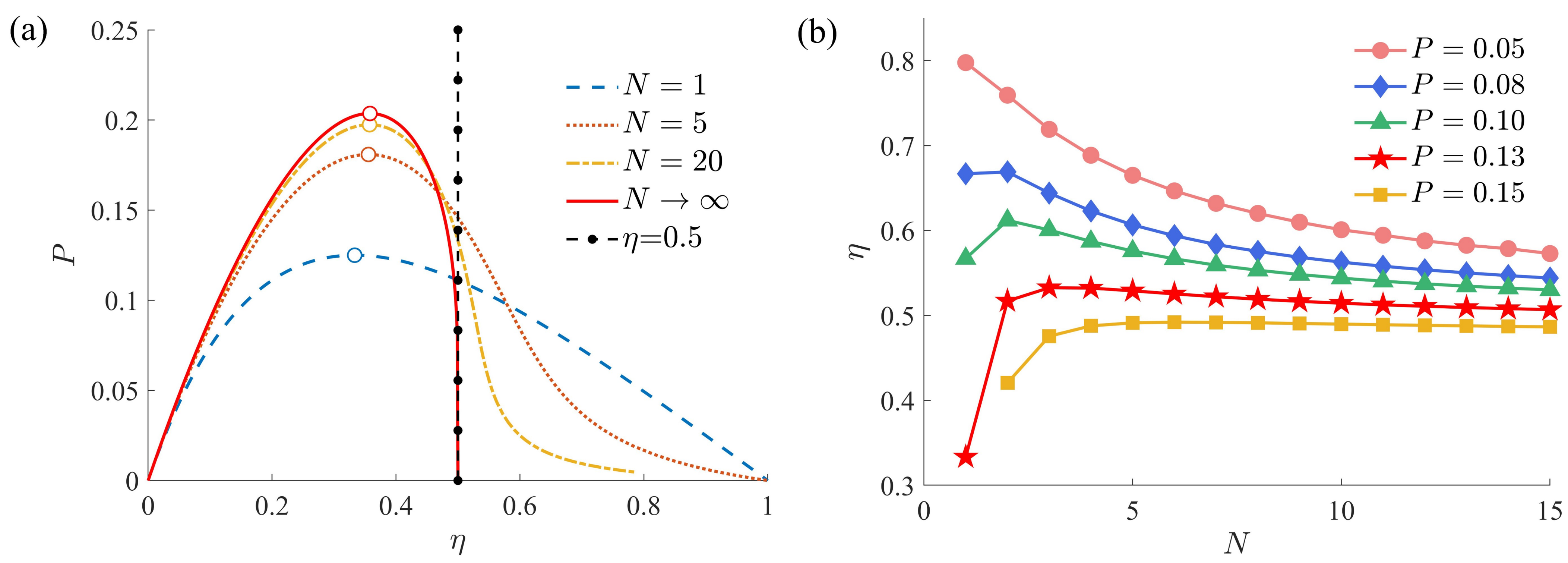

In the regime of $N \gg 1$, retain the first order of $N^{-1}$ of $\tau_{\min}$ as $\tau_{\min} \simeq CR\ln k + CR\frac{\ln^2 k}{2N}$, indicating $1/N$-scaling of decay.

➙ As $N \to \infty$, the lower bound $CR\ln k$ for $\tau_{\min}$ is obtained. MSCC transitions to CV strategy with a relatively high initial current for real-world batteries. It is crucial to consider both charging time indicated by charging power $P$, and charging efficiency $\eta$.

## 4. P-$\eta$ trade-off for MSCC charging

The charging power in the optimal MSCC strategy and the corresponding efficiency:

- $P = \frac{W_{\text{in}}}{\tau_{\min}} = \frac{V_{\text{f}}^2 - V_1^2}{2RN(k^{\frac{1}{N}} - 1)}$
- $\eta = \frac{W_{\text{in}}}{W_{\text{in}} + Q_{\text{diss}}} = \left[1 + 2\frac{(V_{\text{m}} - V_{\text{f}})(k+1)}{(V_{\text{f}} + V_1)(k^{\frac{1}{N}} + 1)}\right]^{-1}$




## Reference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## 5. Summary

✓ The fundamental thermodynamic $P - \eta$ trade-off in typical RC series.
✓ Universal $P - \eta$ trade-off in different real-world charging strategies.
✓ The OCP of MSCC strategy.

**Our study unveils fundamental thermodynamic constraints that serve as stepping stones to understand the physical nature and optimization limit for battery charging.**