Engineering Ratchet-Based Particle Separation via Extended Shortcuts to Isothermality

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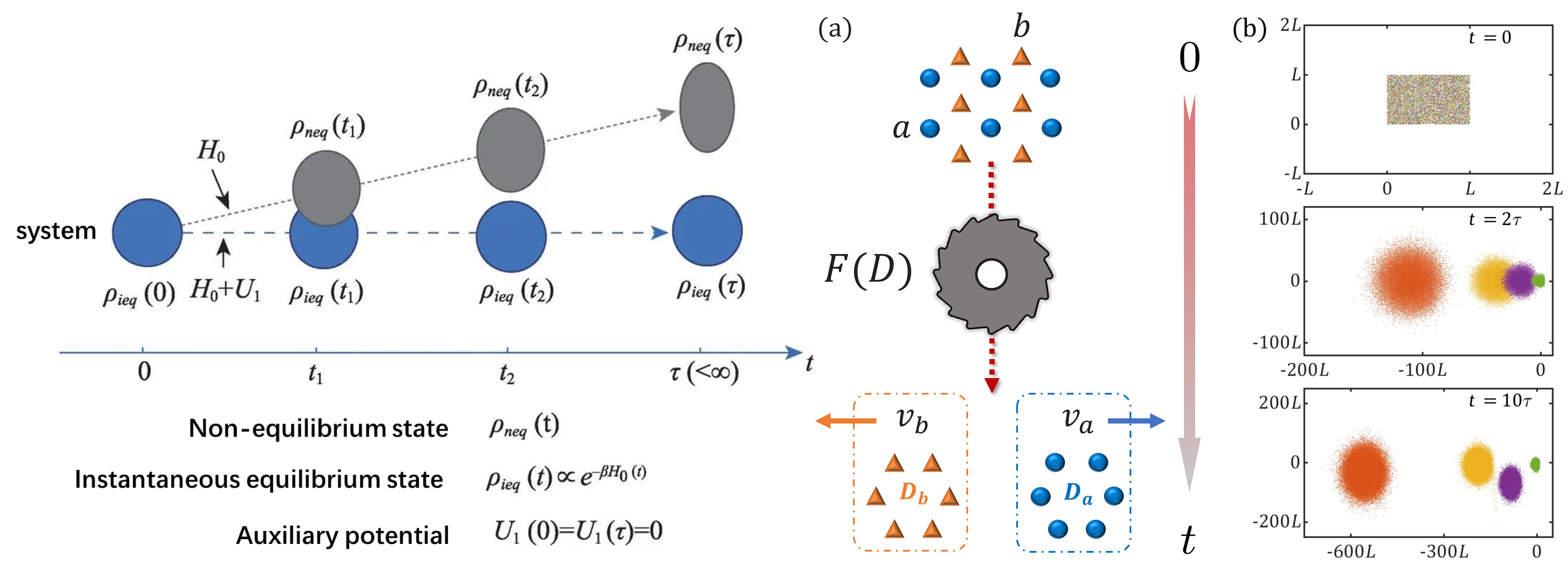
1. Introduction

Microscopic particle separation plays vital role in various scientific and industrial domains. Conventional separation methods relying on external forces or physical barriers inherently exhibit limitations in terms of efficiency, selectivity, and adaptability across diverse particle types. To overcome these limitations, researchers are constantly exploring new separation approaches, among which ratchet-based separation is a noteworthy method. However, in contrast to the extensive numerical studies and experimental investigations on ratchet separation, its theoretical exploration appears weak, particularly lacking in the analysis of energy consumption involved in the separation processes. The latter is of significant importance for achieving energetically efficient separation. **In this work, we propose a non-equilibrium thermodynamic approach to realize controllable separation of overdamped Brownian particles with low energy cost**. This study bridges the gap between thermodynamic process control and particle separation, paving the way for further thermodynamic optimization in ratchet-based particle separation.

2. Theoretical Framework

The **Shortcuts to Isothermality** scheme involves applying an auxiliary potential in the system of interest so that the system remains in the thermal equilibrium distribution of its original Hamiltonian during the driving process. **Our theoretical framework utilize this scheme in the particle separation task, allowing the particles to move under control by consuming extra energy**

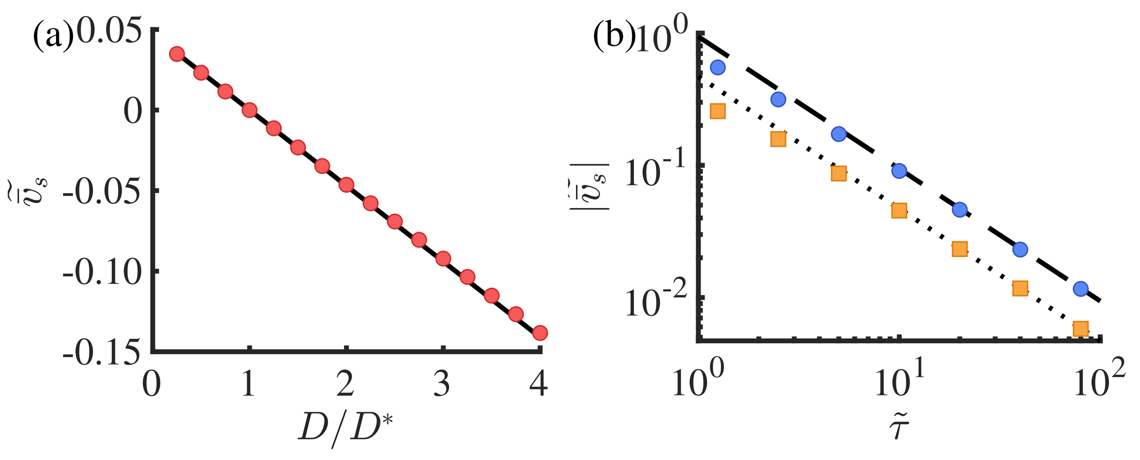


3 Particle Current

$$J_s = \frac{1}{\tau}\left(1-\frac{D}{D^*}\right)\Phi_{\rm rev} - \frac{1}{\tau}\frac{D}{D^*}\oint_I d\lambda\cdot j(\lambda) \quad (1)$$

Integrated flow of reversible ratchets

$$\Phi_{\rm rev} \equiv \oint_I d\lambda \cdot \frac{\int_0^L e^{\beta U_o(x,\lambda)}\nabla_\lambda \int_0^x \rho_o(x',\lambda)dx'dx}{\int_0^L e^{\beta U_o(x,\lambda)}dx} \quad (2)$$



The particle velocity $\tilde{v}_s$ as a function of D/D^* with $\tau = 20$(a); τ for $D/D^* = 2$ (circles) and $D/D^* = 1/2$ (squares) (b)

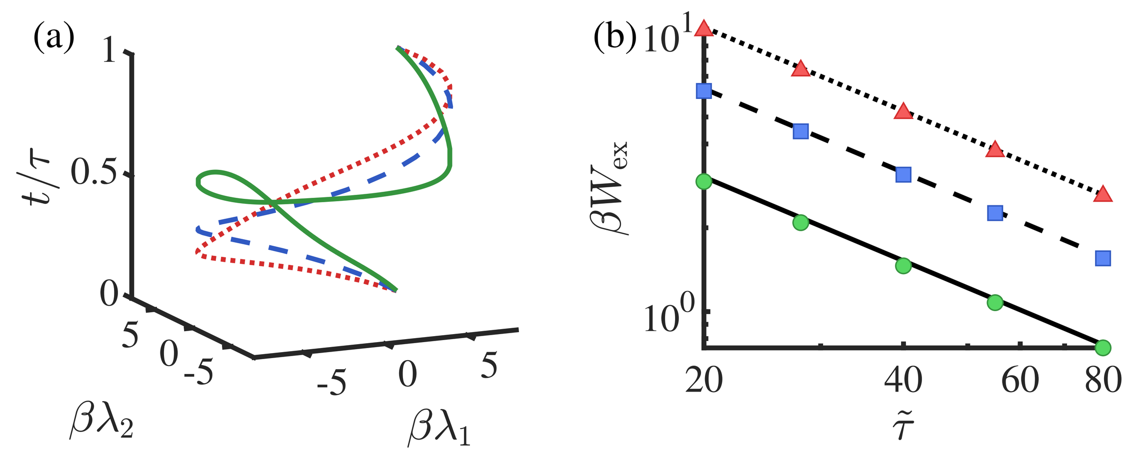
4. Energetic Cost

$$W \ge \Delta E + \frac{D^*\mathcal{L}^2 J_s}{(D^*-D)\Phi_{\rm rev}} \quad (3)$$

Optimal control protocol

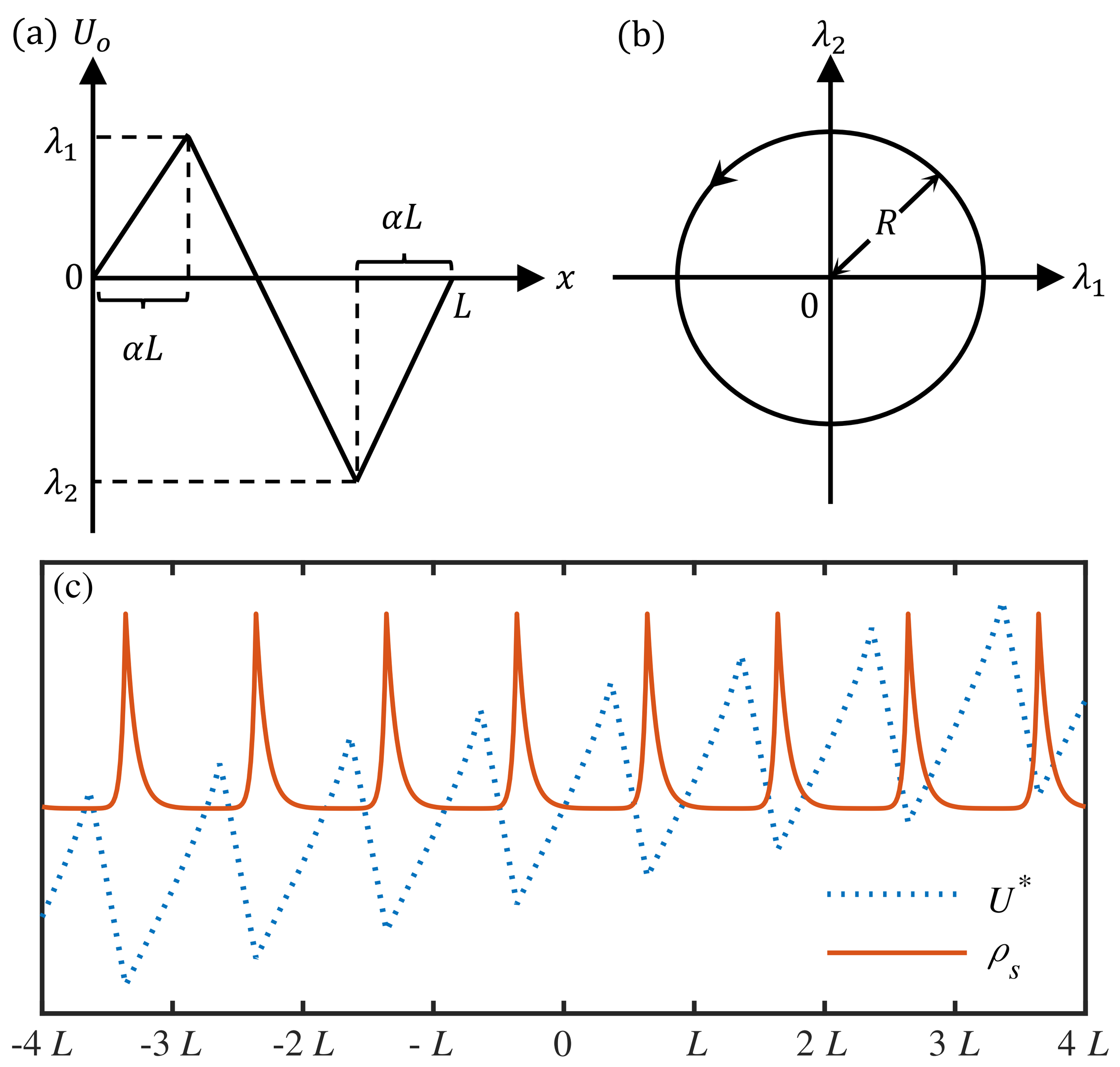
$$\tau\sqrt{\lambda_\alpha\lambda_\beta G_{\alpha\beta}(\lambda)} - \mathcal{L} = 0, \quad (4)$$

$\mathcal{L}$: thermodynamic length

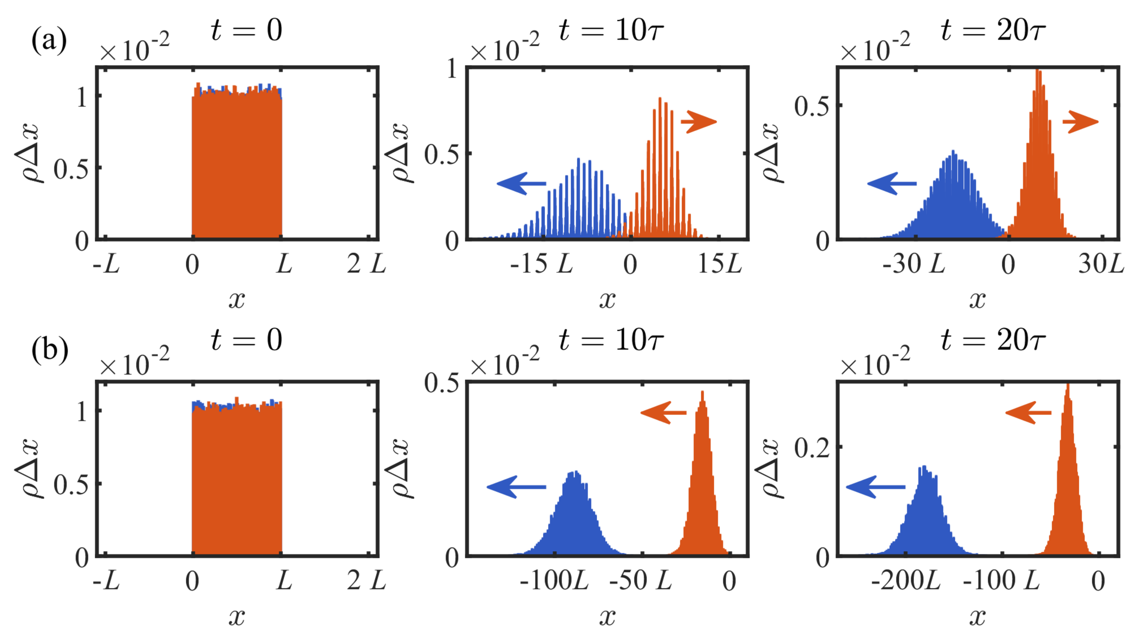


(a) Three different driving protocols; (b) The extra energetic cost $W_{\rm ex}$ as a function of time with $D/D^* = 2$.

5. Particle Separation



(a) Original periodic potential $U_o(x,\lambda)$. (b) The trajectory of the driving protocol $\lambda(t)$. (c) The tilted total potential $U^*(x,t)$ and the periodic steady reduced probability density $\rho_s(x,t)$



Probability distributions of particles with $D/D^* = 2$ (blue lines) and $D/D^* = 1/2$ (orange lines) driven under different $j(\lambda)$

6. Conclusions

✧ **Key findings:**

i) We incorporate shortcut to isothermolity to achieve adjustable particle separation

ii) We derive the particle flux, induced by a diffusion-coefficient-dependent effective force, facilitating particle separation in either the same or opposite directions

iii) we determine the optimal driving protocol that minimizes energetic consumption

✧ **Further exploration:**

i) Theoretical analysis of the fast-driving regime

ii) Modify ratchet potential with alternative thermodynamic control strategies

iii) Compare the performance of different separation methods

References

[1] **Xiu-Hua Zhao, Z. C. Tu and Yu-Han Ma, *Engineering ratchet-based particle separation via extended shortcuts to isothermality*, Phys. Rev. E 110 (2024) 034105**

[2] G. Li,H. T. Quan, and Z. C. Tu, *Shortcuts to isothermality and nonequilibrium work relations*, Phys. Rev. E 96, 012144 (2017).

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